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## **Faculty Working Papers**

THE IMPORTANT ECONOMIC IMPLICATIONS OF LEARNING-BY-DOING FOR POPULATION SIZE AND GROWTH

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#### Summary

Arrow's (and Kaldor's) representation of learning-by-doing and the current state of technique as a function of capital rather than of cumulated output-the latter being the empirical basis for the learning-by-doing concept-leads to major confusion. An obvious though not-very-important example is that a constant production level implies no progress in such a formulation, though there is solid evidence that learning-by-doing continues as cumulated output increases.

The most important implication of learning-by-doing for population policy—with respect to immigration as well as natural increase—is that a population of larger size implies faster growth of consumption per head than does a smaller population size. Arrow's formulation does not reveal this implication, and empirical work built on Arrow's model does not reach this conclusion. The paper unravels an apparent paradox and shows that a corrected formulation of the learning-by-doing model does indeed have this important implication, which is consistent with relevant empirical data of several kinds.

### Acknowledgment

I appreciate helpful readings of this paper by Mark Browning and Jonathan Cave.



# THE IMPORTANT ECONOMIC IMPLICATIONS OF LEARNING-BY-DOING FOR POPULATION SIZE AND GROWTH\*

### Julian L. Simon

### I. INTRODUCTION

Following on the empirical studies of learning-by-doing by Wright (1936), Asher (1956) and others, and the summary and interpretation by Hirsch (1956) and Alchian (1963), Arrow combined the effect of experience with a macromodel to study the economic implications. This model followed Kaldor in making technical progress endogenous, though Kaldor's point of departure was Verdoorn's macro and industry data (1949), rather than the firm-level data on which Arrow built and which lend themselves more obviously to a learning interpretation than do Verdoorn's data.

This note contends that Arrow did not derive the most important implications of learning-by-doing, and that he also reached some wrong conclusions, because he modeled learning-by-doing poorly. These conclusions—that population growth, and even more importantly, population size, have positive effects upon the rate of economic growth through their positive effects on the rate of technical progress—have major implications for the understanding of, and social policy with respect to, population size and growth.

Section II criticizes the concept of experience Arrow used, to make clear the necessity for a fresh start. Section III shows the implications for evaluation of different <u>sizes</u> of population, which requires unraveling an apparent paradox in Arrow's (and Kaldor's)

<sup>\*</sup>I appreciate helpful readings of this paper by Mark Browning and Jonathan Cave.



analyses which leads to incorrect conclusions about this matter. Section IV shows the implications of learning-by-doing for evaluation of the effects on consumption of different rates of population growth on size. Section V summarizes the work and the conclusions.

# II. CRITICISM OF ARROW'S MEASURE OF EXPERIENCE This section is in the nature of clearing of debris.

All the empirical studies on which Arrow relied, and also most of those that have come afterwards (with exceptions to be noted below) related something like "the amount of direct labor required to produce an airframe and the [cumulative] number of airframes produced" (Alchian, 1963, p. 680).

(1) 
$$H_N = H_1 N^{-b} = H_1 - H_1 (1 - N^{-b})$$

where N = serial number of a particular unit of output

 $H_{N}$  = hours of work per unit of output for serial number N h = constant.

Arrow is very clear on this: "The economic examples given above suggest the possibility of using cumulative output (the total of output from the beginning of time) as an index of experience" (p. 157).

But then, because Arrow doubted the psychological generalization implicit in that measurement (which also avoids the necessity of coming up with a number for cumulated output since the beginning of time), he shifted to a very different concept, with entirely different implications. "I therefore take instead cumulative gross investment [the capital stock] (cumulative production of capital goods) as an index of experience" (p. 157). That is,



(2) 
$$A_t = \frac{1}{H_t} = f(K_t) = dK_t^b$$
,

where  $A_{+} = level of technique$ 

K = capital stock

d = constant.

He also shifted from a serial-number concept to a time-period concept, a difficult matter which will be taken up later.

Let us be clear on the possible confusions caused by taking the capital stock rather than cumulative gross output as the index of experience.

One such confusion is that, even using Arrow's own assumptions, cumulative capital and cumulated output yield different conclusions, as may be seen in the following: In monetary terms, the capital/output ratio may be taken (and is so assumed by Arrow) to be roughly constant in this context. Hence the capital stock and one year's production are roughly proportional. So the capital stock can be considered as a proxy for incremental output rather than cumulated output. And incremental output is generally not proportional to cumulated output; if production in t and t+l are equal, cumulative output is higher in t+l than in t even though capital is constant.

Formally,

(3) 
$$A_{t} = a \begin{pmatrix} t \\ \sum_{i=-\infty} Y_{i} \end{pmatrix} b$$

where  $Y_t = output$  in period t

which can be rewritten

(4) 
$$\frac{A_{t} - A_{t-1}}{A_{t-1}} = \frac{\begin{pmatrix} t \\ \Sigma \\ \underline{1} = -\infty \end{pmatrix}^{b} - \begin{pmatrix} t-1 \\ \Sigma \\ \underline{1} = -\infty \end{pmatrix}^{b}}{\begin{pmatrix} t-1 \\ \Sigma \\ \underline{Y}_{\underline{i}} \end{pmatrix}^{b}}$$



where the right-hand side is always positive.

Now consider Arrow's function (2) from which can be written

(2a) 
$$\frac{A_{t} - A_{t-1}}{A_{t-1}} = \frac{K_{t}^{b} - K_{t-1}^{b}}{K_{t-1}^{b}}.$$

If the capital/output ratio is a constant c, as Arrow assumes it is, then (2a) can be rewritten

(5) 
$$\frac{A_{t} - A_{t-1}}{A_{t-1}} = \frac{(cY)_{t}^{b} - (cY)_{t-1}^{b}}{(cY)_{t-1}^{b}} = \frac{Y_{t}^{b} - Y_{t-1}^{b}}{Y_{t-1}^{b}}.$$

This is unequal to (4), as can be seen from the fact that if  $Y_{t-1} = Y_t$ , (4) will be positive but (5) will be zero. And (4) corresponds to the facts that Arrow cites, including the Horndal effect, whereas (5) does not.

Even worse are all the difficulties that flow from that most troublesome of all economic concepts to conceptualize and measure satisfactorily,
capital. Most fundamental here is that the capital concept used in the
model is physical, whereas the observed capital-output ratio that Arrow
builds on is a monetary notion, and there is no necessary or likely correspondence between the two; more about that later.

Arrow's definition of the capital stock also is not analogous to cumulated output in that it excludes old capital that has already obsolesced. This is important because it means that Arrow's model does not sum up experience since the very beginning of the learning process, which causes a paradox in interpretation of rates of productivity change that must be unraveled below.



(Even cumulative output would have to be defined carefully because of the difficulty of estimating it for an economy or society as a whole. The entire transposition from, on the one hand, the firm or industry level with technical measurements to, on the other hand, the national economy with value measurements can—and in this case does—cause the argument to lose its way, and later to be misinterpreted in empirical work by others.)

In brief, the use of capital rather than cumulated output is one source of erroneous interpretation of the implications of learning-by-doing, though not the central issue here.

### III. LEARNING-BY-DOING AND LABOR FORCE SIZE

This section is the piece de resistance of the paper, both because of the necessary subtlety of the argument and because of the greater relevance for policy of the conclusion about population size than of conclusions about golden age growth patterns. It boils down to a distinction between equilibrium and nonequilibrium analyses, with the latter being appropriate for the question under discussion here.

Let us now return to the firm and industry level at which the learning-by-doing phenomenon is actually observed, to investigate the phenomenon's implications for various sizes of the labor force and population.

Here we must be crystal-clear about the nature of the empirical process on which the function is based, because a subtle shift in interpretation has caused a fundamental misinterpretation. For the first unit made, the measured number of hours spent is equal to  $\mathbf{H}_1$  (the constant a in Arrow's notation). The process is assumed to start at this point, with  $\mathbf{H}_1$ -one of the two key values in the system--given exogenously



as a result of a variety of unnamed previous experiences. Successive units require the amount of labor in the initial unit less a proportion of that initial amount, the proportion depending on the serial number of the successive unit. This makes clear that to portray a learning-by-doing process one must either know, or be able to estimate from later data, the original level of skill at which this particular process begins.

Assume that the good i, whose cumulative quantity produced up to period t is indexed by serial number  $N_{t,i}$ , is a normal good. It is reasonable to assume that the output of i in any period t is a proportional function of total income

(6) 
$$N_{r} - N_{r-1} = gY$$

and that total income is a proportional function of labor force size

$$(7) Y = hL.$$

where g and h are constants. The volume  $N_t - N_{t-1} = ghL$  produced in t therefore is an increasing function of the population size. (It is all-important here to distinguish between, on the one hand, the volume produced in a given period t, and, on the other hand, the serial number N, which might be produced in any period t.)

Consider the moment when a given production process N begins with serial number N = 1. For convenience, this takes place on January 1 of year t = 1 in country Alpha. Assume population size is  $P_{t=1}$ , labor force size is  $L_{t=1}$ , and total national output for that year is  $Y_{t=1}$ . For those conditions, there will be X units of good i produced, so at the end of the first year the serial number is  $N_{t=2}^{\alpha} = X$ .



Now assume instead that we are in country Beta and population size instead is  $2P_{t=1}$ , labor force size is  $2L_{t=1}$ , and consequently total national output is  $2Y_{t=1}$ . (Furthermore, the population and output have always been twice as large in Beta as in Alpha). From (6) and (7) we can expect that the end-of-year serial number will be  $N_{t=2}^{\beta} = 2X$  rather than  $N_{t=2}^{\alpha} = X$ . Hence  $N_{t=2}^{\alpha} - N_{t=1}^{\alpha} = X$  for  $L_{t=1}$  and 2X for  $2L_{t=1}$ .

Now consider the rate of change of productivity A over the first period. Whatever the percentage change  $\frac{A_{t=2}-A_{t=1}}{A_{t=1}}$ , the rate will be roughly twice as large for  $2L_{t=1}$  as for  $L_{t=1}$  (assuming that the change is small relative to the initial value, which one can ensure by making the period short, say a day rather than a year).

To elucidate this point, let us write out the learning-by-doing concept in the greatest possible detail. Define the level of productivity

A as the inverse of work time per unit, and using (1)

(1a) 
$$A_{N} = \frac{A_{N=1}}{N-b} = A_{N=1}N^{b}$$
$$= A_{N=1} + A_{N=1}(N^{b} - 1).$$

From (la) it is quite clear that for two firms or economies  $\alpha$  and  $\beta$  that produce, up to the same date,  $N^{\alpha}$  and  $N^{\beta} > N^{\alpha}$  units, the rate of change of productivity from the beginning to that date is greater for  $\beta$  than for  $\alpha$ ,

$$\left(\frac{A_N \beta_{>N} \alpha - A_{N=1}}{A_{N=1}}\right) > \left(\frac{A_N \alpha - A_{N=1}}{A_{N=1}}\right).$$

In Arrow's model we do <u>not</u> find that the rate of change of productivity is a function of output or labor force in anywhere near the same way, because it is in the nature of any capital stock that it does



not correspond in any simple additive way to all the capital produced since the beginning of time. And this is even more true in Arrow's model which includes obsolescence, and an arbitrary obsolescence rule at that.

Even more important, Arrow's model invites one to write an experience function such as

(5) 
$$\frac{A_{t+1} - A_t}{A_t} = \frac{aK_{t+1}^b - aK_t^b}{aK_t}$$

where the dates include any given period. Though Arrow does not write this function explicitly, he likens his model to Kaldor's, who writes the similar function

(9) 
$$\frac{A_{t+1} - A_t}{A_t} = c \left( \frac{K_t - K_{t-1}}{K_{t-1}} \right)$$
.

In Arrow's words, "The production assumptions of this section are designed to play the role assigned by Kaldor to his 'technical progress function,' which relates the rate of growth of output per worker to the rate of growth of capital per worker. I prefer to think of relations between rates of growth as themselves derived from more fundamental relations between the magnitudes involved" (p. 160). He also says that "Verdoorn had also developed a similar simple model" (p. 160); Verdoorn differs in using output per year, but his independent variable is also a rate of change of the <u>current</u> magnitude. So there is little doubt that function (8) is a fair representation of Arrow's model. And in his empirical "Tests of the 'Learning By Doing' Hypothesis," Sheshinski (1967) estimated exactly this model where "the increase in experience ... is attributed to current gross investment" (that is, the rate of change of



productivity = investment ÷ capital stock),\* or in an alternative model, "the change in experience is attributed to the rate of output" (that is, the rate of change of productivity = output ÷ "cumulated gross output," where Sheshinki's empirical referent of the latter concept is unclear but is certainly not cumulated gross output since the beginning of the process, as required for a reasonable portrayal of the learning-by-doing process).

Given that a country's situation at any given date is compared to its own situation at some slightly earlier date, the rate of change of the capital stock may be expected to be the same for larger country Beta as for smaller country Alpha, ceteris paribus. Hence in Arrow's model the rate of increase of productivity is essentially independent of the scale of the economy and of the population, though it will reflect short-run changes in output (through differences in investment and the capital stock). That is, though this model may lend itself to estimating the rate for a given process, it does not enable one to compare the results for two entities of different sizes.

To make the point as graphic as possible: If fathers cut their male offspring's hair monthly, a father with twin sons will get twice as much practice in any period, and his skill at any date will be higher, than that of a father with only one son (assuming equal beginning skill). This does not imply that in each period the twins' father's rate of increase of skill will be higher than that of the father of only one son;

<sup>\*</sup>Actually, Sheshinski estimated Kaldor's model in which the r.h.s. is  $c \sqrt{\frac{K_t - K_{t-1}}{K_{t-1}}}$  rather than Arrow's model in which the right hand side would be  $\sqrt{\frac{K_t^b - K_{t-1}^b}{K_t^b}},$  though he said that he was testing the learning-by-doing model.



if the learning curve has the same exponent at all points, the rates will be the same in each period after the first. Nor need the absolute increase in skill be greater in any period for the twins' father than for the single-son's father. But the rate of increase of productivity from the beginning to any given date will be greater for the father with twins than for the father with one son.\*

There is an apparent paradox here: If two countries of different sizes of population and output are compared for a period starting after a process has begun, at the same date in both, the rates of change of productivity over the period can be the same, even though if the period begins at the beginning of the process, the rate of change of productivity of the larger country will be greater. For example, assume that Alpha produces one unit per period, and Beta produces two units per period. Between periods four and five, Alpha goes from serial 4 to serial 5, whereas Beta goes from 8 to 10. If  $A_1$ —the level of technology of the first unit, the inverse of the numbers of hours required to make the first unit—is equal to 10, if b = .5, and hence if  $A_N$  =  $10N^{-5}$ , then for Alpha

$$\frac{A_{t=5} - A_{t=4}}{A_{t=4}} = \frac{A_{N=5} - A_{N=4}}{A_{N=4}} = \frac{10\sqrt{5} - 10\sqrt{4}}{10\sqrt{4}} \approx .414$$

and also for Beta

$$\frac{A_{t=5} - A_{t=4}}{A_{t=4}} = \frac{A_{N=10} - A_{N=8}}{A_{N=8}} = \frac{10\sqrt{10} - 10\sqrt{8}}{10\sqrt{8}} \approx .414.$$

<sup>\*</sup>If one worries about the inequality in total employment of the two fathers, the example could be changed to a society with one father-barber versus a society with two father-barbers, all with one child each. In the economy with two fathers, they exchange information on what they learn, and hence increase the rate of their learning; this is quite realistic.



But from the beginning of the process to the end of period 5, the calculations are for Alpha

$$\frac{A_{t=5} - A_{t=1}}{A_{t=1}} = \frac{10\sqrt{5} - 10\sqrt{1}}{10\sqrt{1}}$$

and for Beta

$$\frac{A_{t=5} - A_{N=1}}{A_{N=1}} = \frac{10\sqrt{10} - 10\sqrt{1}}{10\sqrt{1}}.$$

So Beta's rate of increase of productivity is faster than Alpha's when calculated from the beginning of the process.

We can see the economic importance of this population effect with a simple simulation model, comparing the results for more-developed technology-producing worlds of two sizes, one twice as large as the other. Such a simple model has a Cobb-Douglas production function, a savings function proportional to output (set at the much-lower-than-steady-state value to avoid difficult arguments), and a one percent labor-force growth rate per year.

The model is as follows:

$$Y_{t} = A_{t}L_{t}^{\alpha}K_{t}^{\beta}$$

$$L_{t} = L_{t-1} + L_{t-1}, \quad \ell = .01$$

$$K_{t} = 2L_{t-1} + K_{t-1}$$

Initial values  $A_t = 1.0$ ,  $L_t = 1000$ ,  $K_t = 1000$ ,  $Y_t = 500$ ,  $\alpha = .67$ ,  $\beta = .33$ . An iterative program is used to make investment approximately a function of current-period income rather than prior-period income, so that the computer model would approximate the steady-state analytic model; the results are much the same with and without this refinement, however.



The technological-change function, for both populations, is

$$M_{t} = m(\sum_{0}^{t-1} Y_{t-1})^{b} \qquad b = .2 \text{ or } .3$$

The learning parameter in the model is initially set to produce 1% productivity change per year in the smaller population. And the model for both populations starts with the same cumulated amount of output obtained by jobbing back from the initial year's value, which is unrealistically advantageous to the smaller population; the result is not sensitive to differences in this initial condition, however.

The relevant comparison in a long-run context would seem to be one in which capital per head has achieved the same level in both the worlds. From this assumption it is obvious that the bigger world must have faster growth, because there is no offset to the productivity effect through capital dilution in this set-up (though later we shall see, in the context of different rates of growth, how the tradeoff between capital dilution and productivity effects plays itself out).

The relevant measure of performance in a policy context is a comparison of present values of future income streams under the different population assumptions. Table 1 shows the present values for the two initial populations at various rates of discount. Clearly the outlook for the future standard of living is better with the larger initial population. Of course the differences would be less with a lower learning parameter, and greater with a higher learning parameter.

Table 1



It is clear, then, that a larger population, market, and total output imply faster economic growth due to learning-by-doing. But there is no chance that this phenomenon will be observed if the Kaldor-Arrow model is used, either theoretically as in Arrow's and Kaldor's work, or empirically, as in Sheshinski's work.

Furthermore, this effect implies that the rate of growth of productivity will increase with time, rather than being steady-state growth as Arrow concluded that it is. To show that this is so, we must return our thinking to the company and industry level at which the learning-by-doing phenomenon actually observed, rather than moving directly to the macro level by simple analogy to the firm.

Once a learning-by-doing process begins, the rate of learning may be constant with increasing production volume.\* But another process that is identical in all economic details with the one previously under discussion, except that it comes along later, will operate in an environment of a bigger total market and total output (due in part to the earlier innovation) even if population size remains constant. Therefore sales will be greater if the good is a normal one, and the rate of learning per unit of time will be faster for the later innovation than for the earlier one. To the extent that these innovations are representative of the economy as a whole, then, the rate of increase of productivity for the economy as a whole will increase over time.

<sup>\*</sup>Or it may decline with output (e.g., Barkai and Levhari, 1973; Levhari and Sheshinski, 1973; Baloff, 1966, and references cited therein). Though the rate of learning in a given product situation may decrease, there may still be changes in the processes which restart a high-rate learning process, and increase the overall rate of learning.



An increasing rate of growth of productivity implies that there is not a constant capital-output ratio, as may be deduced immediately from the well-known

(10) 
$$s = \frac{1}{v}(n + A)$$
.

where s = savings ratio

v = output-capital ratio

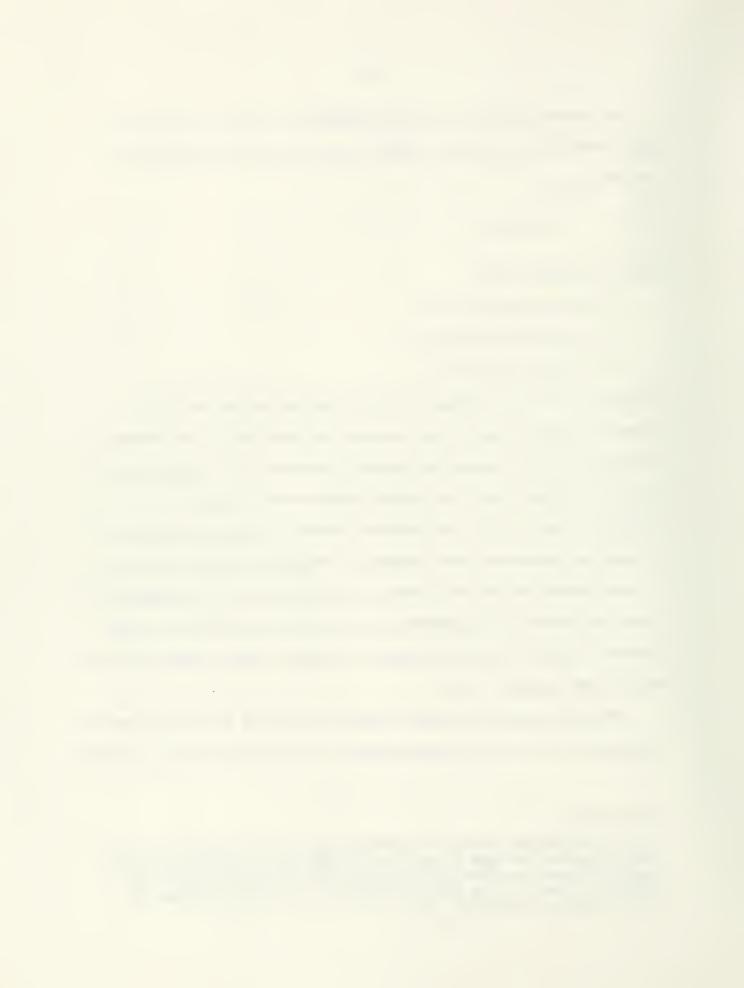
n = population growth rate

A = rate of change of A.

But this is in no way inconsistent with observed reality nor is it a ground for concern, despite the supposed "stylized fact" of an observed constant K/Y ratio, because the monetary estimates of the capital-output ratio tell nothing about the physical capital-output ratio, as I show elsewhere (Simon, 1980). The observed constant K/Y ratio in value terms is simply an inevitable price response of competitive markets, and has no significance for long-run production relationships.\* The relevant K/Y ratio—the physical ratio—probably is not constant in MDC's but rather secularly falling, though this proposition cannot ever be examined rigorously in its aggregate form.

One might wonder why larger economies do not grow faster than smaller economies, in light of the above analysis. In fact, Chenery and associates

<sup>\*</sup>It is still another problem with Arrow's model that despite its real-resource nature, he bases it upon the assumption of a constant K/Y ratio, despite the meaninglessness of the monetary K/Y ratio to describe the physical K/Y ratio in the long-run and the fact that the physical K/Y ratio is falling in the long run.



(e.g., Chenery and Syrquin, 1975) do observe economies of size in crossnational comparisons of growth rates in LDC's. And the Rostas-Clark data
show a strong effect in individual industries (Clark, 1967). But, additionally, to the extent that economies are open, there are forces that
reduce the observed differences in growth among countries. Specialization (e.g., Sweden in autos, Holland in communications) can lead to
larger markets than the national economy provides. And a considerable
proportion of technical progress is not locally tied and moved across
national boundaries. Hence there is no inconsistency between the observed data and the analysis given here. The relevant conceptual unit
for analysis of the sort given above is the Western industrialized world
as a whole, with the relevant comparison being between imagined larger
and smaller sizes of it.

## IV. LEARNING-BY-DOING AND LABOR FORCE GROWTH

It is reasonably easy to show that in a comparison of two populations with different rates of labor-force growth (with optimum savings ratios) that are already on a steady growth path, faster labor-force growth implies higher consumption—as long as technical progress is an increasing function of the size of the labor of on total output, even to the slightest degree. Consider a situation in which technical progress does not depend upon either labor force or total output; if so, technical progress will be the same, and the rate of growth of per-capita output will also be the same, for every rate of population (labor force) growth, though consumption will be lower with higher population growth due to the higher warranted savings rate. But if technical progress is faster with a higher



population growth rate—as it is with function (6a) because faster population growth implies faster increase in aggregate income and hence in capital—then the rate of growth of per capita output must be faster with higher population growth. And this must therefore eventually (no matter how slight the dependence of technical progress on population or output) lead to higher levels of per capita output and consumption. This is the argument in a nutshell.\*

Arrow's model does show increasing returns to scale. But
Arrow does not describe the effect of a change in only L or L. Furthermore, his returns to scale derive from an unusual sort of production
function—one with fixed coefficients rather than a neo-classical form—which would not be appropriate for analysis of the effect of a change in
L or L. And Kaldor, using the same sort of technical progress function
but a different sort of growth model, concludes that an increase in L
depresses the rate of growth of output, or at best leaves it unchanged
(pp. 615-616). More generally, as Eltis puts it, "Both Kaldor and Arrow
wrote long and forceful passages to argue that there is likely to be a
strong endogenous relationship between investment and technical progress
in the real world. Unfortunately, their subsequent mathematical specification of the relationship precluded any endogenous connection in
equilibrium—in their growth models" (1973, p. 154). And this is even

<sup>\*</sup>The comparison of steady growth paths is not the relevant comparison for a given society's policy choice at a given moment, however. Rather, the society wishes to evaluate its future streams of costs and benefits with different rates of population growth, given its present endowment of capital and level of income. That comparison is the subject of another paper.



more true of the connection between the rate of growth of the labor force and the rate of growth of consumption.

In short, learning-by-doing implies that the steady-state consumption path inevitably comes to be higher with faster population growth than with slower population growth, in contrast to standard growth theory's conclusions.

Phelps' model (1966), too, implies that faster labor-force growth produces a higher golden-age consumption path. And Eltis (1973) arrived at this conclusion explicitly. My own model (1977, Chapter 6), which has a technical progress function closer to a learning-by-doing formulation than that of Phelps or Eltis, also produced a conclusion that higher population growth implies higher consumption after an initial period during which capital-dilution produces lower consumption. And a straightforward neo-classical model with Arrow's own formulation of learning-by-doing also produces this result under most assumptions.

Though it is outside the main purpose of this paper, the simulation described earlier was also run with different rates of growth of the labor force, all other initial conditions identical. In the earlier years, the lower rate of population growth (1% yearly) leads to higher per-worker consumption because of the capital dilution effect than does 2% population growth. But after two or three decades (just how long depends on the assumptions) the higher rate of population growth comes to have higher per-worker consumption due to its faster rate of productivity growth. And a present-value computation shows that the higher rate of population growth yields better results up to very significant discount rates, and at all discount rates where there are meaningful differences between the



various population-growth assumptions. These results may be seen in Table 1.

## V. SUMMARY AND CONCLUSIONS

Arrow's (and Kaldor's) representation of learning-by-doing and the current state of technique as a function of capital rather than of cumulated output—the latter being the empirical basis for the learning-by-doing concept—leads to major confusion. An obvious though not-very-important example is that a constant production level implies no progress in such a formulation, though there is solid evidence that learning-by-doing continues as cumulated output increases.

An important implication of the learning-by-doing phenomenon (as with almost any model where technical progress is endogenous) is that it leads to a different conclusion about population growth than does standard growth theory, implying that the eventual consumption path is higher with faster population growth. Arrow's model does not show this clearly, Kaldor somehow reached the opposite conclusion, and standard growth-theoretic texts (e.g., Solow, 1970; Wan, 1971; Brems, 1973; Dixit, 1976)—even the specialized summary of growth theory with respect to population growth of Pitchford (1974)—do not mention this implication.

The most important implication of learning-by-doing for population policy-with respect to immigration as well as natural increase—is that a population of larger <u>size</u> implies faster growth of consumption per head than does a smaller population size. Arrow's formulation does not reveal this implication, and empirical work built on Arrow's model does not reach this conclusion. The paper unravels an apparent paradox and



shows that a corrected formulation of the learning-by-doing model does indeed have this important implication, which is consistent with relevant empirical data of several kinds.



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Table 1

## Present Values (600 Years) of Per-Capita Consumption Streams With Two Different Initial Population Sizes And Various Rates of Savings and Population Growth

b = .5

						1000						2000	
Ĺ	ĸ												
0	0	0%	1	2	3	4	5	0	1	2	3	4	5
0	0	.653 E+3	.732 E+2	.315 E+2	.1497 E+2	.142 E+2	.111 E+2	.869 E+3	.897 E+2	.366 E+2	.221 E+2	.156 E+2	.120 E+2
0	4	.190 E+4	.120 E+3	.403 E+2	.228 E+2	.157 E+2	.119 E+2	.495 E+4	.215 E+3	.566 E+2	.288 E+2	.187 E+2	.137 E+2
1	2	.103 E+5	.179 E+3	.383 E+2	.211 E+2	.146 E+2	.112 E+2	.198 E+5	.295 E+3	.498 E+2	.250 E+2	.166 E+2	.124 E+2
1	4	.191 E+5	.282 E+3	.470 E+2	.236 E+2	.858 E+2	.118 E+2	.372 E+5	.489 E+3	.644 E+2	.290 E+2	.183 E+2	.133 E+2
1	8	.353 E+5	.405 E+3	.616 E+2	.278 E+2	.176 E+2	.128 E+2	.692 E+5	.838 E+3	.897 E+2	.356 E+2	.211 E+2	.147 E+2
2	4	.913 E+6	.448 E+4	.879 E+2	.249 E+2	.155 E+2	.115 E+2	.182 E+7	.888 E+4	.148 E+3	.326 E+2	.185 E+2	.131 E+2
2	8	.173 E+7	.842 E+4	.141 E+3	.313 E+2	.178 E+2	.126 E+2	.346 E+7	.167 E+5	.249 E+3	.436 E+2	.221 E+2	.148 E+2















